Exercises of Derivatives

Math 161 - Fall 2014

1. Use the definition of derivation to the following functions

   (a) $f(x) = x^2 + x + 4$

   (b) $g(t) = \dfrac{1}{t+3}$

   (c) $h(x) = \sqrt{x+4}$

2. Compute the derivative of the following functions and simplify if possible

   (a) $f(x) = x^{33} + 4x^{12} + 3x^4 + \sqrt{42}x$

   (b) $g(y) = \sin y + \cos y + \tan y$

   (c) $h(t) = (t^4 + 6)\sin t$

   (d) $f(s) = \cot s$

   (e) $g(x) = \dfrac{1}{\pi}x^{\pi} - x$

   (f) $h(z) = \dfrac{\sin z}{z^3 + z^2 + 3z + 1}$

   (g) $r(x) = \sqrt{x}(x^3 + 3x + 3)$

   (h) $F(v) = \dfrac{v^3 + 3v^2 + 4}{v}$

   (i) $Q(y) = (1 - y^{-1})^{-1}$

   (j) $R(m) = \sqrt[5]{\dfrac{m^2}{\sec m}}$

   (k) $S(p) = (p^3 + 2p^2 + 4)^5(3p^2 + 4)^3$

   (l) $T(x) = \dfrac{(x-1)(x-4)}{(x-2)(x-3)}$

   (m) $G(z) = \sin 3z \cos 3z$

   (n) $H(t) = t^4 \sin t \cos t$

   (o) $f(x) = \sqrt{\sin\sqrt{x^2+5}}$

   (p) $g(t) = \dfrac{t^3}{\sqrt{(t^4+1)}}$

   (q) $p(v) = (v^3 + v + 4)^5(2v^3 + 3v^2 + 2)^4$

(r) $\phi(x) = \dfrac{1}{\sqrt[3]{x + \sqrt[3]{x}}}$

(s) $\psi(t) = \sin^2\left(\dfrac{t^3+1}{t^2+2t}\right)$

3. Calculate the first and second derivative of the following functions

   (a) $y = \dfrac{x^2 - 2\sqrt{x}}{x}$

   (b) $v = \sqrt[5]{u^3} - 4\sqrt[7]{u^{11}}$

   (c) $z = \cos(x^2)$

4. Find the equation of the tangent line and the normal line to the curve at the given point

   (a) $y = x^2 + x + 3$ at $x = 1$

   (b) $y = \sqrt{1 + x^3}$ at $x = 2$

   (c) $y = 6\cos x$ at $(\frac{\pi}{3}, 3)$

   (d) $y = \sin(\sin(x))$ at $(\pi, 0)$

5. The position of a particle is given by $s(t) = t^3 - 12t + 3$, where t is measured in seconds and s in meters.

   (a) Find the velocity and acceleration functions.

   (b) Determine when the particle is at rest.

6. Find $\dfrac{d^{74}}{dx^{74}}(\sin x)$

7. Find the n-th derivative of $f(x) = x^n$

8. For what values of x does the graph of $f(x) = x^3 - 6x^2 - 15x + 4430$ have a horizontal tangent line?

9. Show that the curve $y = 12x^3 + 10x - 3$ has no tangent line with slope 8.

10. If f ang g are a differentiable functions, find an expression for the derivative of each of the following function

    (a) $y = x^n f(x)$

(b) $y = \dfrac{1 + xf(x)}{\sqrt[3]{x^2}}$

(c) $y = x^3 f(x^5)$

(d) $y = \dfrac{f(x)g(x)}{f(x) + g(x)}$

11. Suppose that $h(x) = f(x)g(x)$ and $F(x) = f(g(x))$, where $f(2) = 3$, $g(2) = 5$, $g^{'}(2) = 4, f^{'}(2) = -2$ and $f^{'}(5) = 11$. Find $a)$ $h^{'}(2)$ and $b) F^{'}(2)$

12. Evaluate the following limits (By expressing the limit as a derivative)

(a) $\lim\limits_{x \to 1} \dfrac{x^{4000} - 1}{x - 1}$

(b) $\lim\limits_{h \to 0} \dfrac{\sqrt[4]{16 + h} - 2}{h}$

13. Find $\dfrac{dy}{dx}$ by implicit differentiation

(a) $y \cos x = x^2 + y^2$

(b) $x \sin y + y \sin x = 1$

(c) $4 \cos x \sin y = 1$

14. Find $y^{''}$ by implicit differentiation

(a) $x^3 + y^3 = 1$

(b) $9x^2 + y^2 = 9$

15. Use implicit differentiation to find an equation of the tangent line to the curve at the given point

(a) $x^2 + 4xy + y^2 = 13,\ (2, 1)$

(b) $2(x^2 + y^2)^2 = 25(x^2 - y^2),\ (3, 1)$

16. The length of a rectangle is increasing at a rate of 8 cm/s and its width is increasing at a rate of 10 cm/s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?

17. Air is being pumped into a spherical balloon so that its volume increases at a rate of $300\,\mathrm{m}^3/\mathrm{hr}$. How fast is the radius of the balloon increasing when the radius is 100 m?

18. Find the linear approximation of the function $g(x) = \sqrt[3]{x+1}$ at $a = 0$ and use it to approximate the numbers $\sqrt[3]{0.95}$ and $\sqrt[3]{1.1}$.